Calculating Radar Cross Sections on a Quantum Computer

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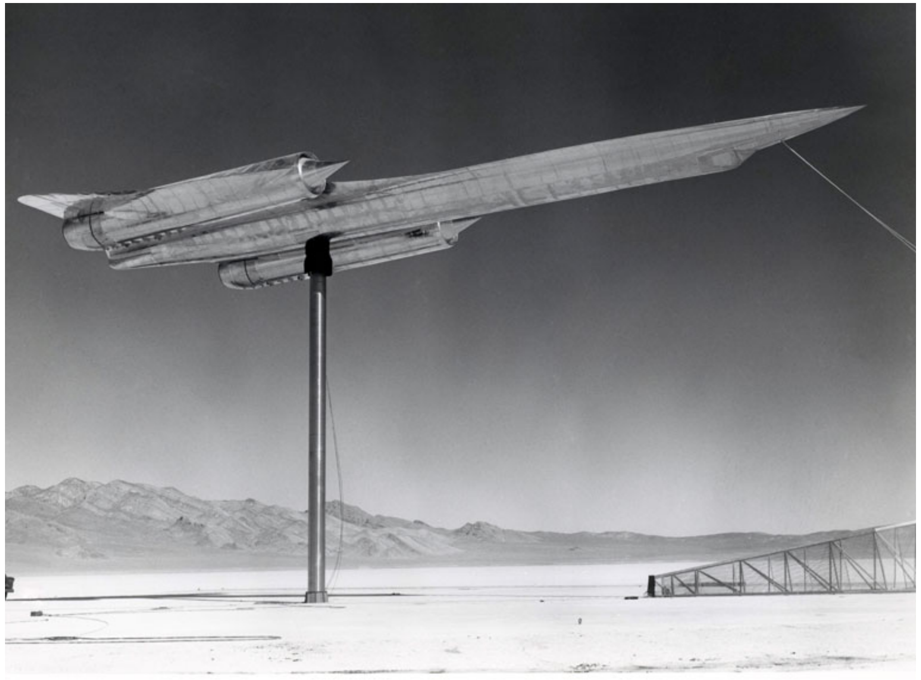
Classical RCS Calculation

- Radar Cross Sections
  - Important aircraft property
  - Electromagnetic scattering
  - Ratio of emitted and received field strength
- RCS Calculation
  - Stationary scenario
  - Solve Maxwell‘s equations
  - Approximate by method of finite-elements
  - Solve large sparse linear system



Quantum RCS Calculation

- HHL Algorithm
  - Solve sparse linear systems
  - Harrow, Hassidim, Lloyd (2008)
  - $O(s^2 k^2 \log(N)\, \varepsilon^{-1})$
    - **Exponential speedup** in matrix dimension
    - Polynomially worse in condition number

- Challenges
  - **Input** and **output** of classical data
    -> Clader et al. 2013
  - Speedup only asymptotically

-> Focus on matrix input

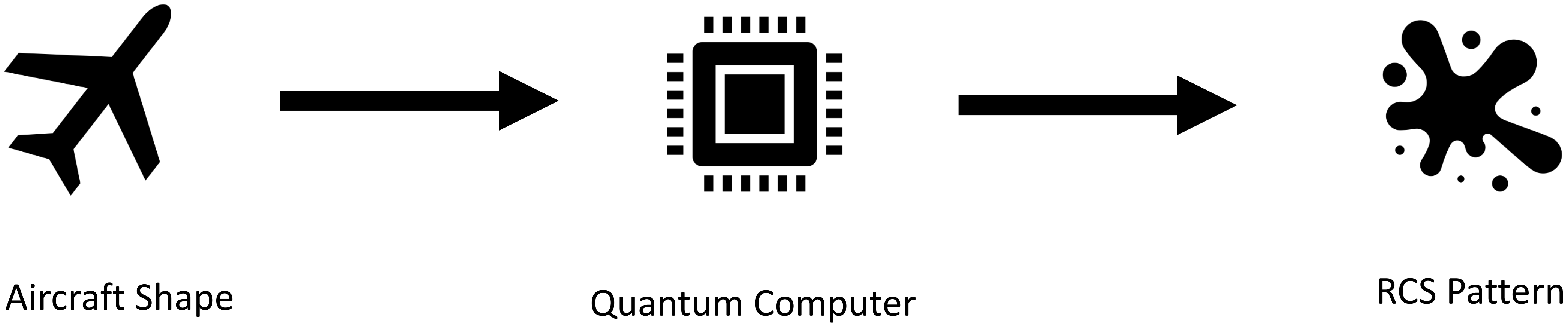
Efficient Matrix Input

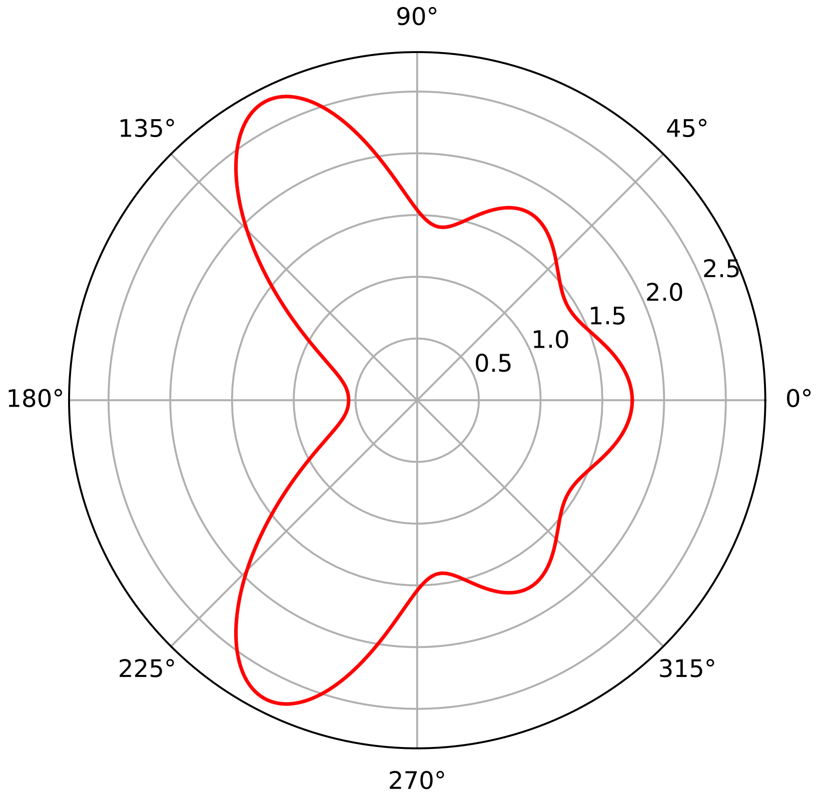
- Classical input not efficient
- QRAM remains open question

- Computation on the fly

  -> Matrix generation i.e. FEM on QC

  -> Scherer et al. 2015





Return of the RCS

- Why revisit the problem?
  - Research interest for DLR
  - Unclear implications of design decisions
  - New insights
- Evaluated design decisions
  - Automatic vs. manual classical resource estimation
  - Fixed vs. floating point arithmetics
  - Advanced arithmetic functions

Automatic vs. Manual Estimation

- Low-level classical resource estimate
  - Reimplementation in Python
  - Count elementary operations
  - Compare to high-level quipper estimate

-> Similar results to Scherer et al.

Fixed Point vs. Floating Point Arithmetics

- T-gate count as resource metric
- Floating Point implementation as presented by Soeken et al.

- Precision Ranking: FP16, FP32, QP32, FP64

-> Comparable resource requirements

OP	ADD	SUB	MUL	DIV	CMP	SUM
Count	378	72	536	55	79	1120
Fixed 32 / QP32	7,64E+03	7,64E+03	**1,45E+05**	9,59E+05	2,07E+03	**1,34E+08**
Float 16 / FP16	1,75E+04	1,73E+04	4,78E+04	2,11E+04	5,10E+03	3,46E+07
Float 32 / FP32	4,12E+04	3,94E+04	1,26E+05	6,63E+04	9,26E+03	8,93E+07
Float 64 / FP64	8,58E+04	8,52E+04	**4,33E+05**	2,60E+05	1,76E+04	**2,84E+08**

Trigonometric Functions

$$\arctan(x) \approx \begin{cases} -\frac{\pi}{2} - \frac{x}{x^2+0.28}, & \text{for } x < -1 \\ \frac{1}{1+0.28x^2}, & \text{for } -1 < x < 1 \\ \frac{\pi}{2} - \frac{x}{x^2+0.28}, & \text{for } 1 < x. \end{cases}$$

- „Arctan“ subroutine dominates requirements
- Originally approximated by CORDIC method
- Approximate by real function up to 0.005rad

-> Improve resource requirements by one order of magnitude

Conclusion

- One of the few applications with a detailed resource estimate
- Design decisions impact requirements

- Resource requirements dramatically exceed midterm possibilites
- Reliable estimate for realization time seems impossible

Main References

1. Quantum Algorithm for Linear Systems of Equations by A. W. Harrow, A. Hassidim and S. Lloyd in 2008 at arXiv (0811.3171)
2. Preconditioned Quantum Linear System Algorithm by B. D. Clader, B. C. Jacobs and C. R. Sprouse in 2013 at arXiv (1301.2340)
3. Concrete Resource Analysis of the Quantum Linear System Algorithm used to compute the Electromagnetic Scattering Cross Section of a 2D Target by A. Scherer, B. Valiron, S.-C. Mau, S. Alexander, E. van den Berg, T. E. Chapuran in 2015 at arXiv (1505.06552)
4. Logic Synthesis for Quantum Computing by M. Soeken, M. Roetteler, N. Wiebe and G. De Micheli in 2017 at arXiv (1706.02721)